

A comprehensive weighted evolving network model^{*}

Chunguang Li^{1†}, Guanrong Chen²

¹Institute of Electronic Systems, School of Electronic Engineering,
University of Electronic Science and Technology of China,
Chengdu, 610054, P. R. China.

²Department of Electronic Engineering, City University of Hong Kong,
83 Tat Chee Avenue, Kowloon, Hong Kong, P. R. China.

Abstract

Many social, technological, biological, and economical systems are best described by weighted networks, whose properties and dynamics depend not only on their structures but also on the connection weights among their nodes. However, most existing research work on complex network models are concentrated on network structures, with connection weights among their nodes being either 1 or 0. In this paper, we propose a new weighted evolving network model. Numerical simulations indicate that this network model yields three power-law distributions of the node degrees, connection weights and node strengths. Particularly, some other properties of the distributions, such as the droop-head and heavy-tail effects, can also be reflected by this model.

Complex networks are currently being studied across many fields of science and engineering [1], stimulated by the fact that many systems in nature can be described by models of complex networks. A complex network is a large set of interconnected nodes, in which a node is a fundamental unit usually with specific dynamical or information contents. Examples include the Internet, which is a complex network of routers and computers connected by various physical or wireless links; the World Wide Web, which is an enormous virtual network of web sites connected by hyperlinks; and various communication networks, food webs, biological neural networks, electrical power grids, social and economic relations, coauthorship and citation networks of scientists, cellular and metabolic networks, etc. The ubiquity of various real and artificial networks naturally motivates the current intensive study of complex networks, on both theory and applications.

Many properties of complex networks have been reported in the current literatures. Notably, it is found that many complex networks show the small-world property [2], which implies that a network has a high degree of clustering as in a regular network and a small average distance between nodes as in a random network. Another significant recent discovery is the observation that many large-scale complex networks are scale-free. This means that the degree distributions of these complex networks follow a power law form $P(k) \sim k^{-\gamma}$ for large k , where $P(k)$ is the probability that a node in the network is connected to k other nodes

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[†]Corresponding author, Email: cgli@uestc.edu.cn

and γ is a positive real number determined by the given network. Since power laws are free of characteristic scale, such networks are called “scale-free networks” [3]. The scale-free nature of many real-world networks can be generated by a mechanism of growing with preferential attachment [3].

In most growing network models, all the links are binary with values being either 1 or 0. However, many real-world networks, such as various scientific collaboration networks [4] and ecosystems [5], display different interaction weights between nodes. Recently, some weighted evolving network models are proposed. In [6], a weighted network model was presented, in which both the structure and the connection weights are driven by the connectivity according to the preferential attachment rule. In [7], four weighted evolving network models were proposed and analyzed. In [8], a weighted evolving network model with stochastic weight assignments was suggested. More recently, additional information about the connection weights in real-world complex networks has been found. In [9], for example, we found that the connection weights of many real-world networks indeed obey power-law distributions. In [10], it was also found that the strengths of nodes in complex networks obey a power-law distribution. Here, the strength s_i of node i is defined as [10]

$$s_i = \sum_{j \in V(i)} w_{ij} \quad (1)$$

where the sum runs over the set $V(i)$ of some specified neighbors of node i . Thus, together with the power-law distribution of degrees, there are three power-law distributions concerning the structure and the connection in a complex network according to statistical studies. The weighted network models considered in the aforementioned references reflect only part of these three power-law distributions.

It should also be noted that, most existing weighted network models are driven entirely by the preferential attachment scheme in structure and the preferential strengthening scheme in connection weights. In real-world networks, other than these kinds of “preference”, there are also “randomness”. For example, in scientific collaboration networks, if a new comer is a student to enter the field, he/she usually collaborates with his/her advisor other than a popular author unknown to him/her in this field. In this case, the nodes that new nodes attached to are likely being uniformly distributed. This can be described by *random attachment*. Moreover, if two individuals collaborate successfully many times, they are likely to keep on collaborating and even increase their joint work. This is the effect of *preferential strengthening*. But, sometimes, this preferential strengthening is disobedient. For example, if an individual moved to a new university, he/she is most likely to begin to collaborate with new colleagues in the same field, and stop or reduce the collaboration with his/her original co-authors. This can be described by *random strengthening*. In this paper, we propose a new weighted evolving network model, in which we consider both the preferential effects and the random properties of complex networks. This network model displays all the three power-law distributions mentioned above. Besides, some other distribution properties of networks, such as the droop-head (in the foreside of the power-law distribution, data are usually located below the declining line with a slope $-\gamma$) and heavy-tail (in the endside of the power-law distribution, data are usually widely spread out) effects can also be reflected by this new network model. Existing network models generally cannot reflect these properties. Recently, in [11], the authors explained the “droop-head” shape of $P(k)$ distributions in complex networks by using nonextensive entropy approach. But it seems that no explicit connection exists between their approach and our network model.

For simplicity, we only consider undirected network models. Directed weighted evolving network model will be studied separately. The proposed model is defined by the following scheme:

Step 1. Start from a small number m_0 ($m_0 > 1$) of fully connected nodes.

Step 2. At each time step, pick a preferred probability $\alpha \in [0, 1]$ and, with a uniform distribution, randomly generate a real number $s \in [0, 1]$. If $s \leq \alpha$, then a new node of strength 1 is added with probability α , and this new node is connected to an existing node which is selected from among all the existing nodes. This existing node, to which the new node is connected, is selected according to the rule specified at Step 3 below. But if $s > \alpha$, then no new node will be added. Instead, with probability $1 - \alpha$, a new connection with weight 1 is added between two existing nodes, where these two existing nodes are selected according to the rule specified at Step 4 below.

Step 3. Pick a preferred probability $\beta \in [0, 1]$ and, with a uniform distribution, randomly generate a real number $r \in [0, 1]$. If $r \leq \beta$, then with probability β an existing node is selected with the following probability:

$$\Pi(k_i) = \frac{k_i}{\sum_j k_j} \quad (2)$$

where k_i is the degree of node i (*preferential attachment*); but if $r > \beta$, then with probability $1 - \beta$ an existing node is selected randomly (*random attachment*). (Note: this selected node is the “existing node that the new added node is connected to” discussed in Step 2 above.)

Step 4. Pick a preferred probability $\eta \in [0, 1]$ and, with a uniform distribution, randomly generate a real number $\xi \in [0, 1]$. If $\xi \leq \eta$, then with probability η two existing nodes are chosen from among all existing ones, with the following probability:

$$\Pi(i, j) = \frac{w_{ij}}{\sum_{k,l} w_{kl}} \quad (3)$$

where w_{ij} ($= w_{ji}$) is the connection weight value between nodes i and j (*preferential strengthening*); but if $\xi > \eta$, then with probability $1 - \eta$ two existing nodes are chosen randomly (*random strengthening*).

After N time steps, this scheme generates a network with $m_0 + N\alpha$ nodes in the sense of mathematical expectation and the sum of total connection weights is $\frac{1}{2} \sum_{i,j} w_{ij} = N + \frac{1}{2}m_0!$. Parameter α controls the ratio of growing and strengthening; β and η control the ratios between “preference” and “randomness”. In reality, different networks have different values of these parameters, specified by the nature of the given network.

We performed numerical simulations with different parameter values of α, β, η and N . Here, we show the simulation results with $m_0 = 3$ and probabilities $\alpha = 0.3, \beta = \eta = 0.8$. First, we study the degree distribution $P(k)$, which is defined as the probability that a randomly selected node has degree k (a node has k connections). In Fig.1, we show that the degree distribution $P(k)$ obeys a power-law distribution $k^{-\gamma}$ for a large range of k with exponent $\gamma \approx 2.8$, which has a droop-head and a heavy-tail. This is consistent with the statistical results of many real-world networks [3]. In Fig.2, we show the connection weight distribution $P(w)$, which is defined as the probability that a randomly selected link between two nodes has a weight value w . As can be seen from Fig.2, this distribution also behaves in a power-law form, with exponent $\gamma \approx 3$, which is also similar to many real-world data [9]. In Fig.3, we show the node strength distribution $P(s)$,

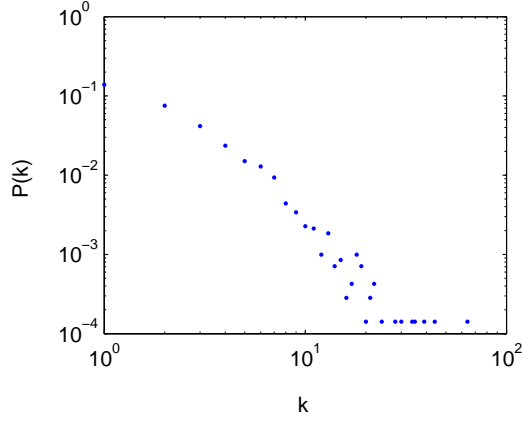


Figure 1: Probability distribution $P(k)$ of degree k ($m_0 = 3, N = 8000, \alpha = 0.3, \beta = \eta = 0.8$).

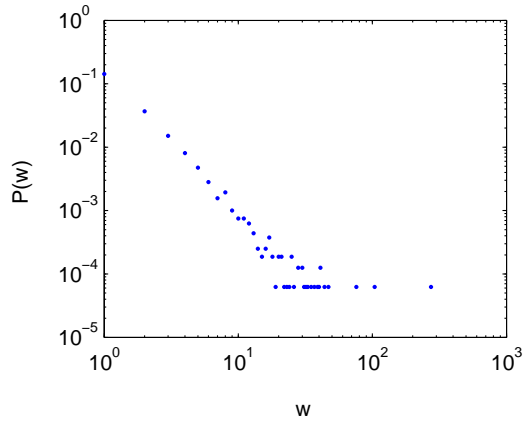


Figure 2: Probability distribution $P(w)$ of the connection weight w ($m_0 = 3, N = 8000, \alpha = 0.3, \beta = \eta = 0.8$).

which is defined as the probability that a randomly selected node has a strength value s . This distribution also obeys a power-law form for a large range of strength values, with exponent $\gamma \approx 2.1$. It also has a droop-head and a heavy-tail, which again coincides with the statistical results of many real-world networks shown in [10]. In Fig.4, we show the time evolution of the strengths of three initial nodes. As can be seen, the increasing speeds of the strength curves decrease gradually as t goes on. In real-world networks, this is also the case due to limited resources and competition etc.

The above simulation results show that our new network model can indeed mimics many real-world complex networks comprehensively. For networks with non-integer connection weight values, such as neural networks, we can also mimic them by the proposed model, using proper non-integer constants as the weight values in each step of the evolving process.

Recently, we noticed that in [12] the authors gave a complex network model with both preferential and random attachments. However the network model considered in [12] is not a weighted network model, and the evolving scheme is fundamentally different from ours.

In summary, to reflect the growing and strengthening dynamics with “preference” and “randomness”, we have proposed a comprehensive weighted evolving network model. Simulation results show that this

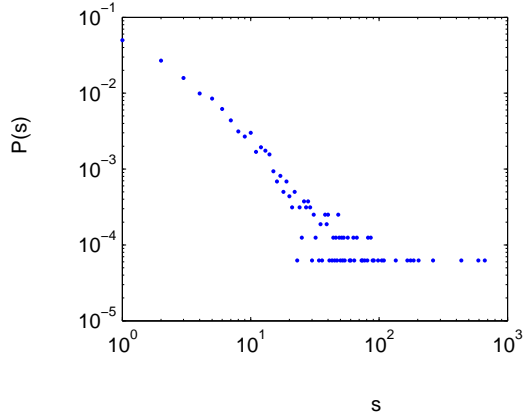


Figure 3: Probability distribution $P(s)$ of the node strength s ($m_0 = 3, N = 8000, \alpha = 0.3, \beta = \eta = 0.8$).

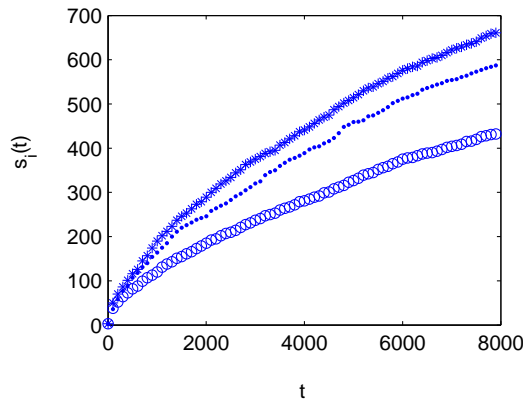


Figure 4: Time evolution of the strengths of three initial nodes.

model can displays all the three power-law distributions regarding the network structure and connection strengths observed in statistical studies of many real-world networks. Also, the droop-head and heavy-tail properties of these distributions, which are observed in many real-world networks, can be reflected by this new network model. In comparison, existing evolving network models generally cannot describe all these features together. Thus, in this paper, we have provided an effective model to mimic many real-world weighted evolving networks.

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